Title: Solving Quadratic Equations by Completing the Square
Class: Math 107
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Instructions to tutor: Read instructions and follow all steps for each problem exactly as given.
Keywords/Tags: Quadratic, equation, square root, completing square, solution

Solving Quadratic Equations by Completing the Square

Purpose:
This is intended to refresh your knowledge about solving quadratic equations by completing the square.

Recall that a **quadratic equation** is an equation that can be written in the form $ax^2 + bx + c = 0$, with $a \neq 0$. For example, $3x^2 + 4x - 7 = 0$, $6 - x^2 = 2x$, and $x(x+6) = 14$ are all quadratic equations. Note that the second two equations would require a couple algebraic steps to be put into the form shown above.

We have seen previously that we can solve the equation $(x+2)^2 = 7$ by taking the square root of each side to obtain $x = -2 \pm \sqrt{7}$. What made this problem quick to solve was the **perfect square** $(x+2)^2$.

If we can get a perfect square in any quadratic equation, so that it looked similar to the above, then we would be in pretty good shape – we would be able to solve every quadratic equation. Fortunately there is a method for building perfect squares, called *completing the square.*

Completing the Square
To make $x^2 + bx$ a perfect square, we must add the term $\left(\frac{b}{2}\right)^2$. Then $x^2 + bx + \left(\frac{b}{2}\right)^2 = (x + \frac{b}{2})^2$.

Example: Complete the square for $x^2 + 12x$.

First note that $b = 12$. So $\left(\dfrac{b}{2}\right)^2 = \left(\dfrac{12}{2}\right)^2 = 6^2 = 36$.

If we add this to $x^2 + 12x$, we should be able to factor it as a perfect square:

$$x^2 + 12x + 36 = (x+6)(x+6) = (x+6)^2$$

Example: Now it's your turn. Complete the square for $x^2 - 8x$.

First $b =$ ______ .

So we need to add $\left(\frac{b}{2}\right)^2 =$ ______ to $x^2 - 8x$ in order to build a perfect square.

Now go ahead and build the perfect square and factor it.

Did you obtain $(x-4)^2$? Good!

Now we need to see how this can aid us in solving quadratic equations.

Example: Solve $x^2 + 12x = 5$ by completing the square.

As we saw previously, we must add 36 to $x^2 + 12x$ in order to build a perfect square. So we will add 36 to each side of the equation.

This gives $x^2 + 12x = 5 \quad \Rightarrow \quad x^2 + 12x + 36 = 5 + 36 \quad \Rightarrow \quad (x+6)^2 = 41$.

Now we can use the square-root method to finish the problem.

$$(x+6)^2 = 41 \quad \Rightarrow \quad \sqrt{(x+6)^2} = \pm\sqrt{41} \quad \Rightarrow \quad x + 6 = \pm\sqrt{41} \quad \Rightarrow \quad x = -6 \pm \sqrt{41}$$

Example: Now it's your turn. Solve $x^2 - 8x - 3 = 0$ by completing the square.

First you should add 3 to both sides to make this look like the previous example.

Next you need to add the value obtained in the first example on this page to both sides.

Do you now have $(x-4)^2 = 19$? Good! Now finish the problem using square roots.

Did you obtain the solutions $x = 4 \pm \sqrt{19}$. Good! Now you are on your own for the next page.

1. Solve each quadratic equation by completing the square.

(a) $x^2 - 8x + 12 = 0$

(b) $x^2 + 6x + 7 = 0$

(c) $2x^2 + 4x - 10 = 0$ (For this one, you should first add 10 to each side, and then divide both sides by 2 to obtain $x^2 + 2x$ on the left hand side.)

Check your answers – If you did not get these, consult a tutor for help.

1. (a) $x = 2, 6$ (b) $x = -3 \pm \sqrt{2}$ (c) $x = -1 \pm \sqrt{6}$